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Abstract

The concept of cross-correlation has been developed in two distinct fields: signal processing and statistics. In the area of signal processing, the cross-correlation function can be used to transform one or more signals so that they can be viewed with an altered perspective. For instance, cross-correlation functions can be used to produce plots that make it easier to identify hidden signals within the data. Cross-correlation functions provide the basis for many more sophisticated signal-processing procedures as well. Digital imaging techniques also rely heavily on cross-correlation procedures, but these methods are not covered in the chapter. In the realm of statistics, cross-correlation functions provide a measure of association between signals. The Pearson product-moment correlation coefficient is simply a normalized version of a cross-correlation. When two times series data sets are cross-correlated, a measure of temporal similarity is achieved. The cross-correlation function in its simplest form is easy to use and quiet intuitive. This chapter builds on simple cross-correlation procedures to illustrate the wide variety of uses they have in the field of biomechanics and to give the reader an intuitive feel for some more complicated analysis procedures. Concepts from both signal processing and statistics are discussed, and the procedures are applied to several practical problems.

Disciplines

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Comments

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CHAPTER 7

Time Series Analysis: The Cross-Correlation Function

Timothy R. Derrick, PhD, and Joshua M. Thomas, MS

The concept of cross-correlation has been developed in two distinct fields: signal processing and statistics. In the area of signal processing, the cross-correlation function can be used to transform one or more signals so that they can be viewed with an altered perspective. For instance, cross-correlation functions can be used to produce plots that make it easier to identify hidden signals within the data. Cross-correlation functions provide the basis for many more sophisticated signal-processing procedures as well. Digital imaging techniques also rely heavily on cross-correlation procedures, but these methods are not covered in this chapter. In the realm of statistics, **cross-correlation** functions provide a measure of association between signals. The Pearson product-moment correlation coefficient is simply a normalized version of a cross-correlation. When two time series data sets are cross-correlated, a measure of temporal similarity is achieved. The cross-correlation function in its simplest form is easy to use and quite intuitive. This chapter builds on simple cross-correlation procedures to illustrate the wide variety of uses they have in the field of biomechanics and to give the reader an intuitive feel for some more complicated analysis procedures. Concepts from both signal processing and statistics are discussed, and the procedures are applied to several practical problems.

In any discussion of the analysis of data, it is important to be specific about their nature. Biomechanists typically collect time series data that are discrete, equally spaced, and stochastic. *Time series* data change as a function of time. *Discrete* data have been digitized at specific time periods from the continuous biological signal that we are interested in. This chapter assumes that sampling theory restrictions on the digitization process have not been violated (Hamill, Caldwell, and Derrick, 1997). *Equally spaced* indicates that the time between successive digitizations is constant. **Stochastic** refers to the fact that even though successive digitizations of the signal are dependent, they are only partly determined by past values. This is in contrast to a **deterministic** signal, in which case future data points can be exactly predicted from past data points.

Time Series Analyses

Modern technologies allow biomechanists to collect vast quantities of data in a relatively short period of time. Biological signals such as positions, velocities, accelerations, forces, joint angles, and **electromyography** can be collected at high rates for increasingly longer periods of time. Mass storage devices are still strained to the limits, as they were decades ago, but they are capable of storing thousands of times more data. Rather than letting this space go to waste, we collect additional data to gain a more holistic view of the activity. The days of carrying the data from a biomechanics research project on a floppy disk are gone, and at times a compact disk is now inadequate. This plethora of data presents organizational problems as well as analytical problems.

Although descriptive techniques for analyzing time series data may appear to be basic, they are important for understanding and verifying the quality of the data. The first step in any analysis should be to visually inspect the data. Viewing the raw numbers is usually not practical, but we can rapidly inspect the data in graphical form. Graphing time series data allows one to observe trends and variations, as well as to observe any outliers that are inconsistent with the rest of the data. The outlier may be a true variation in performance or may be an error caused by malfunctioning equipment or unintended movement patterns. It is important to understand one's data in order to make appropriate adjustments.

Once one has an accurate set of curves that one wants to compare, the choice is between three modes of time series analysis typically used by biomechanists. First, pertinent discrete points on the curve could be identified, and their magnitude and/or the time at which the points occur could be noted. Second, the entire curve could be used to calculate a variable such as the average. Third, the curve could be transformed into a different curve, after which one of these three analyses could be applied again. An example is a **differentiation** transformation that one would use to calculate velocity from position data. The change in position is divided by the change in time, and the resultant curve indicates the rate of change of position (or velocity). Peak velocities, time to peak velocities, average velocities, and so on could then be found.

If the timing of the peak value is important, then the time from initiation to the peak value could be recorded, either as an absolute or as a relative value. Relative values may be expressed as a percentage of a complete cycle or a portion of a cycle. For instance, the time to the peak vertical ground reaction force during running is often expressed as a percentage of the stance phase. Other variables could be calculated from the vertical ground reaction force. Differentiating a force curve gives an indication of the rate of loading, while integration results in the calculation of the impulse.

Discrete point analysis is certainly not the only way to analyze curves. In fact, there can be a vast amount of information that is ignored during a discrete point analysis. Other techniques analyze the entire curve by condensing it into a single number. For example, rather than finding the maximum value of a particular curve, one could calculate the average value over the entire sampling period. Thus, the entire curve is represented by a single value.

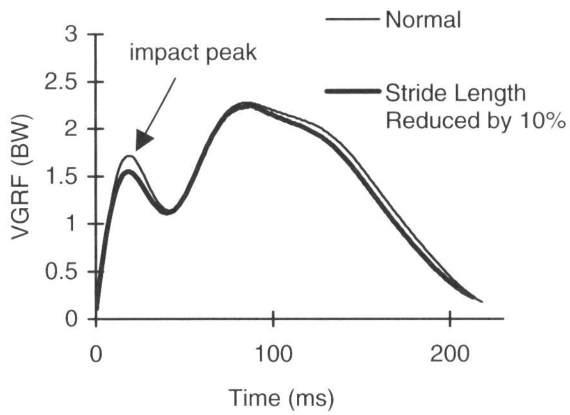


Figure 7.1 Vertical ground reaction forces (VGRF) at different stride lengths. There is a 9.4% decrease in the impact peak when stride length is reduced by 10%, and there is a 2.8% decrease in the average force. The units are body weights (BW).

Data from Derrick, Caldwell, and Hamill, 2000.

Discrete point analysis has some advantages over whole-curve analysis. It allows the researcher to focus on the pertinent portion of the curve. For instance, if one is considering the potential for injury, it may be useful to know the peak forces involved rather than the average forces. Information at points other than the maximum value may not be important. Figure 7.1 shows the effect of a 10% reduction in stride length on the vertical ground reaction forces during running (Derrick, Caldwell, and Hamill, 2000). The peak impact force decreases from 1.71 body weights (BW) during normal running to 1.55 BW with the reduced stride length. This is a 9.4% reduction in the peak impact force. On the other hand, if the force is averaged over the complete **stance phase,** the value decreases from 1.46 BW to 1.42 BW. This is a reduction of only 2.8%. The portions of the curve other than the peak impact may act to dilute the differences.

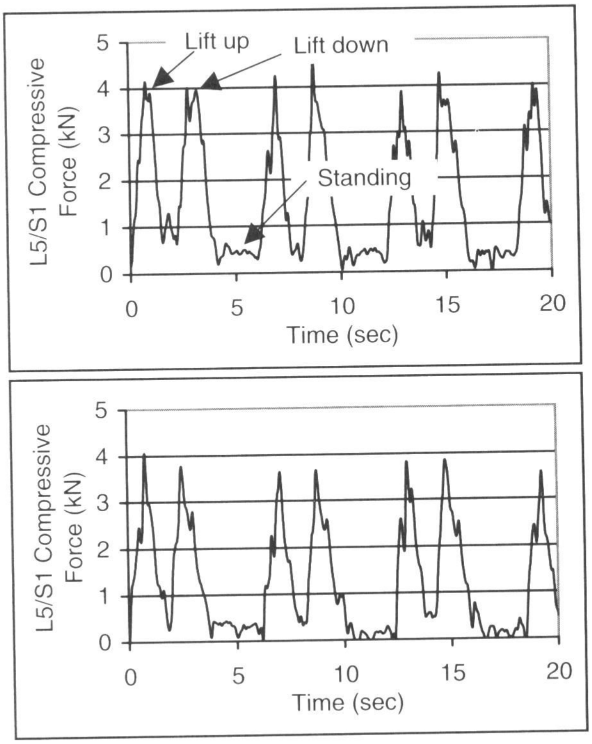


Figure 7.2 Pre- and postfatigue compressive forces. The graphs show L5/S1 compressive forces during lifting of a 10-kg crate multiple times *(a)* prefatigue and *(b)* postfatigue. The peak compressive forces decrease 11.3%, and the average forces decrease 18.9%.

In some instances the information before or after the peak values may be important. Figure 7.2 shows the bone-on-bone compressive forces between the fifth lumbar and first sacral vertebrae during lifting of a crate before and after the back muscles became fatigued. The peak compressive force decreased 11.3% after the subject was fatigued (from 2,057 N to 1,826 N), while the average compressive force decreased by 18.9% (from 734 N to 595 N). Some of the difference between pre- and postfatigue occurs when the subject is standing between lifts. It is up to the researcher to determine which method of assessment is more suitable, but a discrete point analysis may not always be the most appropriate technique.

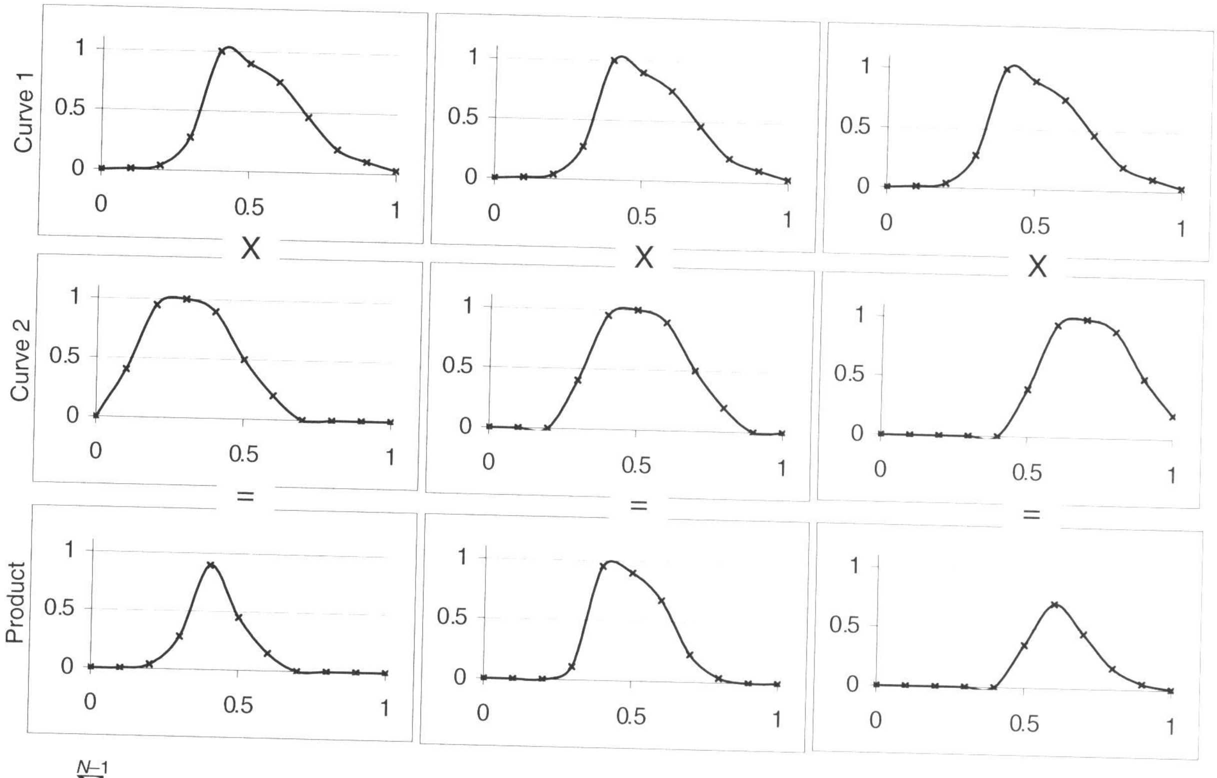
Defining the Cross-Correlation Function

Cross-correlation is a method by which the degree of similarity between two sets of numbers can be quantified. The process involves two entire curves so that information between peak values is assessed. The procedure is very simple, yet the general concept is used in a variety of advanced analysis techniques. These techniques are all based on the fact that if one carries out a point-by-point multiplication of two data sets, the sum of the products will be a quantification of their relationship (equation 7.1).

$$r_{xy} = \sum_{i=0}^{N-1} x_i y_i \tag{7.1}$$

where N is the number of data points in each data series, x_i is the ith data point of the first data series, y_i is the ith data point of the second data series, and r_{xy} is the correlation.

Readers familiar with matrix notation may note that this cross-correlation is simply the dot product of vectors *x* and *y*. Two time series curves are presented in figure 7.3. Curve 1 does not change, while curve 2 is time shifted in each of the three columns in the figure. Cross-correlation values are given at the bottom of each column. The middle column shows the greatest degree of linear relationship between curve 1 and curve 2. The curves tend to rise and fall at the same time. The cross-correlation value of 2.91 quantifies this similarity. Time shifting curve 2 to the left or to the right tends to reduce the cross-correlation (1.82 and 1.77, respectively). Subtracting the mean value of each time series has the effect of accentuating the cross-correlation values because times at which the curves have the opposite sign reduce the cross-correlation value. For instance, the cross-correlations calculated after removal of the mean values for the three sets of curves in figure 7.3 are 0.47, 1.55, and 0.41 for the left-shifted, normal, and right-shifted curves, respectively. Subtracting the mean value also gives significance to a negative cross-correlation. If the mean values are subtracted, a negative correlation indicates that the two time series have an inverse relationship. As one curve is increasing, the other is decreasing. If the mean values are not subtracted, a negative cross-correlation can be obtained if one of the curves is positive and the other is negative, even if there is a perfect relationship.



	Column 1	Column 2	Column 3
$r_{xy} = \sum_{i=0}^{N-1} x_i y_i =$	1.82	2.91	1.77
$r_{xy} = \sum_{i=0}^{N-1} (x_i / \bar{x})(y_i / \bar{y}) =$	0.47	1.55	0.41

Figure 7.3 The effects of time series shifts on the cross-correlation. Numerical values at the bottom of each column represent the cross-correlation of curve 1 and curve 2 without and with removal of the mean values of each curve. The bottom row of curves shows the result of multiplying curve 1 and curve 2.

Example 7.1

Time	Curve 1	Curve 2
0.00	0.00	0.00
0.10	0.01	0.40
0.20	0.04	0.95
0.30	0.28	1.00
0.40	1.00	0.90
0.50	0.90	0.50
0.60	0.75	0.20
0.70	0.46	0.00
0.80	0.20	0.00
0.90	0.10	0.00
1.00	0.03	0.00

Curve 1 × Curve 2
0.00
0.00
0.04
0.28
0.90
0.45
0.15
0.00
0.00
0.00
0.00
Sum 1.82

Use the values for curve 1 and curve 2 shown in the table to calculate the correlation using equation 7.1.

It is a common practice to shift one of the curves relative to the other as was done with curve 2 of figure 7.3. The number of data points that the signal is shifted is called the **lag** and is denoted by ℓ.

$$r_{xy}(\ell) = \sum_{i=0}^{N-1} x_i y_i \tag{7.2}$$

Two issues arise with use of this form of the cross-correlation function. Shifting the second of two curves toward the right leaves an unmatched data point at the start of the first curve and another at the end of the second curve. These unmatched data points can then be matched with each other ("wrapped"), or the unmatched data points can be ignored. For instance, if we shift {1,3,5,7} one data point to the right relative to {4,6,8,10}, the matched pairs are 1-6, 3-8, and 5-10. If we wrap the data, we create another matched pair from the last point of the first series and first point of the second series (7-4). If we do not wrap the data, then the 7 and the 4 are ignored and we are left with a shortened series.

The data should be wrapped only if they are circular. For instance, if an activity such as steady state cycling was being investigated and data were collected from top dead center (TDC) of one stroke to TDC of the next stroke of the same leg (a complete cycle), then the data could be considered circular and wrapping may be appropriate. On the other hand, if the stance phase of a running cycle was being studied, the data would represent only a portion of a complete cycle and it would be inappropriate to wrap the data. If the unmatched data points are ignored, then the number of data points that are being cross-correlated will become lower with increasing lags, and this will tend to reduce the sum of the products (equation 7.2) and reduce the number of data points that the correlation is based on.

The second issue arises because the cross-correlation given by equation 7.2 is not unitless. The cross-correlation depends on the units of x and y, and therefore it is difficult to compare cross-correlations from different data sets. In order to prevent a reduction in the sum of the products with increasing lags and to make the cross-correlation unitless, most applications "normalize" equation 7.2 by dividing it by the square root of the product of the autocorrelation of x at zero lag and the square root of the autocorrelation of y at zero lag (equation 7.3).

$$\rho_{xy}(\ell) = \frac{r_{xy}(\ell)}{\sqrt{r_{xx}(0)}\sqrt{r_{yy}(0)}} \tag{7.3}$$

The autocorrelation is simply a data series cross-correlated with itself. The autocorrelation of the data series x and a lag of zero would be written $r_{xx}(0)$. Since the denominator represents perfect cross-correlations, it is always greater than the numerator. Thus the value for ρ will never exceed 1.0. When the mean values are subtracted (equation 7.4), this normalized cross-correlation will become negative only when the curves have an inverse relationship. This is the version of the cross-correlation that is most commonly used.

$$\rho_{xy}(\ell) = \frac{\sum_{i=0}^{N-1}\left(x_i - \bar{x}\right) * \left(y_{i-\ell} - \bar{y}\right)}{\sqrt{\sum_{i=0}^{N-1}\left(x_i - \bar{x}\right)^2}\sqrt{\sum_{i=0}^{N-1}\left(y_{i-\ell} - \bar{y}\right)^2}} \tag{7.4}$$

Statisticians will recognize equation 7.4 as the Pearson product-moment correlation coefficient. It is often described as the degree to which x and y vary together divided by the degree to which x and y vary separately. Even with use of the normalized cross-correlation formula, correlations should not be calculated when the lag approaches N unless the data are wrapped. If the data are not wrapped the correlations will be based on too few data points as ℓ increases. For instance, if two 25-point series are cross-correlated with a lag of 20, there will be 20 unmatched numbers in each series. This leaves a correlation that is based on only 20% of the original data. A good rule of thumb is to calculate lags only up to $N/2$. In practice, it is often the case that only lags of short duration are of interest.

Pearson Product-Moment Correlations

The researcher may desire a simple measure of the temporal similarity of two curves. The Pearson product-moment correlation provides such a measure (Derrick, Bates, and Dufek, 1994). This correlation is usually performed with a zero lag; but if appropriate, the peak correlation can also be analyzed when multiple lags are available. Figure 7.4 shows two time series curves along with a scatter plot. The scatter plot

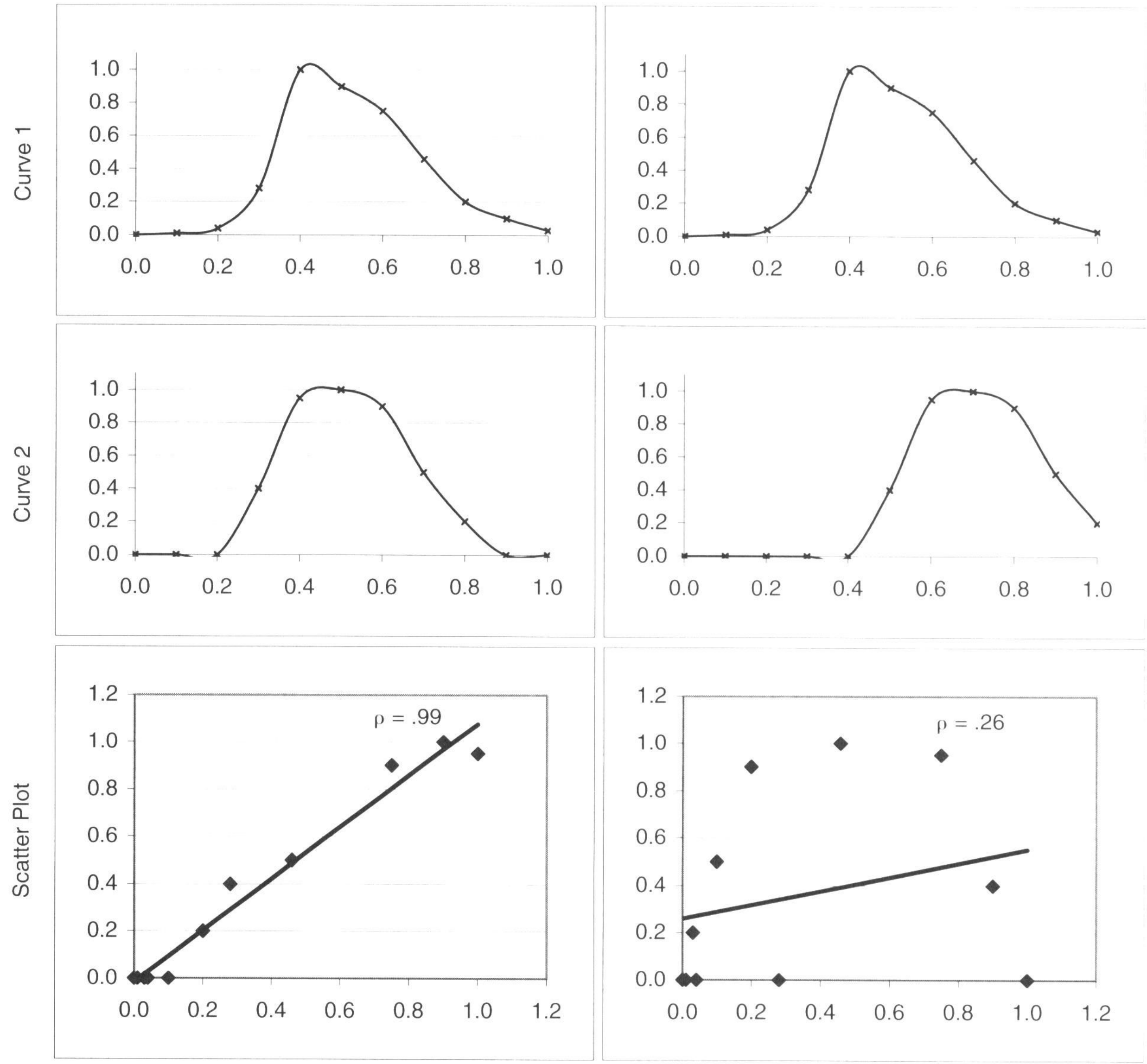


Figure 7.4 The relationship of the Pearson product-moment correlation to temporal similarity. Scatter plots show the relationship between the time series curves. The correlation coefficient ρ quantifies the degree of relationship. Curve 1 and curve 2 have good temporal similarity in the left column, but curve 2 has been shifted in the right column in order to portray a disruption in the timing.

shows curve 1 plotted on the horizontal axis with curve 2 plotted on the vertical axis. The linear regression line is also overlaid on the plot. The correlation can be thought of as a measure of how closely the data points line up on the linear regression line. Changes in one of the curves will be matched by changes in the other if the relationship is strong. All of the points will line up on the regression line if the relationship is perfect, and the correlation will be 1.0. The curves from figure 7.3 are reproduced in figure 7.4. In the left column, the peaks occur at the same time and the correlation is .99. The right column shows the relationship when curve 2 is shifted in time so that the peaks no longer line up. The correlation value is reduced to .26. This illustrates the use of the Pearson product-moment correlation as a measure of the temporal similarity of two curves.

Figure 7.5 shows two pairs of curves along with the scatter plots and correlation values. Both sets of curves show a relationship in which the curves begin at zero, rise to peaks that occur at the same time, and then return to the same amplitude. The

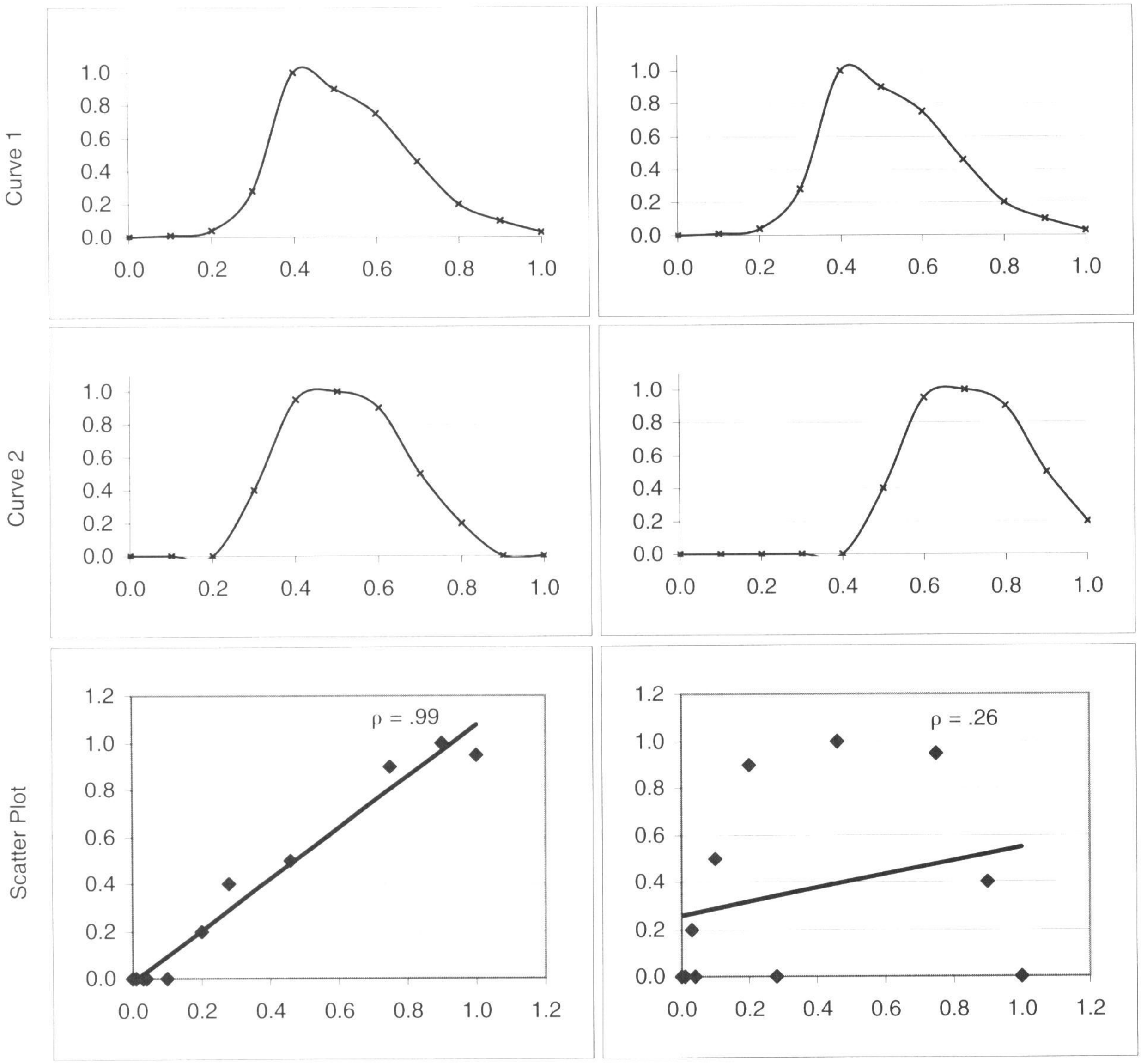


Figure 7.5 Reductions in the correlation with altered endpoints. Time series curves and scatter plots demonstrate how changing the relationship in a part of the curve affects the correlation coefficient (ρ). The left column shows a perfect relationship (curve 1 is a scaled version of curve 2). The right column shows what happens to this relationship if the curves do not end at the same level they start at.

main difference is that the left-hand curves both return to zero while the right-hand curves return to a value somewhat above zero. Although this seems to be an innocuous difference, it does cause the cross-correlation to decrease from 1.00 to .97. This drop occurs because the relationship before the peak value is different from the relationship after the peak value. Separately, the portion before the peak value and the portion after the peak value have nearly perfect relationships ($r \approx 1.00$), but together the relationship is less than perfect.

Stergiou, Bates, and James (1999) used the correlation technique to assess disruptions in the timing between the knee and the subtalar joint during running over an obstacle. Normal running at a self-selected pace produced a correlation of .832 between the knee and the **rearfoot angle** curves. This correlation was reduced to .742 when subjects ran over an obstacle that was 15% of their standing height. The obstacle tended to increase the vertical ground reaction force, which resulted in a change to the shape of the rearfoot curve from unimodal to bimodal (one local minimum to two local minimums). The knee angle curve remained unimodal in both conditions. The change produced greater asynchrony between these joints, which may result in a higher incidence of injury. The change was detected using the Pearson product-moment correlation coefficient because changing the rearfoot curve from unimodal to bimodal made the rearfoot and knee curves less similar.

Fisher Z-Transformations

Correlation coefficients are not normally distributed. As the correlation increases, the distribution becomes more negatively skewed. The values can be transformed so that statistics that assume a normal distribution can be calculated. The Fisher Z-transformation (an inverse hyperbolic tangent) has been used to accomplish this (Otnes and Enochson, 1978):

$$z = \tanh^{-1}\left[r_{xy}(\ell)\right] = \frac{1}{2}\ln\frac{1+r_{xy}(\ell)}{1-r_{xy}(\ell)} \tag{7.5}$$

where z is the Fisher Z-transformation, r_{xy} is the correlation coefficient and, ℓ is the lag.

It can be shown that z is approximately normally distributed with variance

$$\sigma_z^2 = \frac{1}{N-2-\ell} \tag{7.6}$$

where σ_z^2 is variance of the z-scores, N is the number of data points, and ℓ is the lag.

Thus a correlation of .75 would have a z-score of 0.97. Taking the hyperbolic tangent of the z-score will reproduce the correlation coefficient. These z-scores can also be used to average multiple correlations when one is analyzing data across trials or across subjects.

Example 7.2

Calculate the z-score for a correlation coefficient of .75.

$$z = \frac{1}{2}\ln\frac{1+r_{xy}(\ell)}{1-r_{xy}(\ell)} = \frac{1}{2}\ln\frac{1+.75}{1-.75} = \frac{1}{2}\ln 7 = \frac{1}{2}(1.946) = 0.97$$

Li and Caldwell (1999) calculated the confidence intervals of the peak cross-correlation to find the **phase shift** between gluteus maximus electromyography (EMG) and the crank angle during cycling. This method allowed the phase shift measure to be objective, in contrast to the traditional subjective threshold identification methods.

The authors calculated the confidence interval (CI) so that standard statistical analysis of the correlation coefficient was possible. They found that the EMG was shifted 20° counterclockwise in the crank cycle as cadence increased from low to high.

Other Measures of Similarity

Other techniques can be used to measure temporal similarity. For instance, it may be desirable to simply measure the percentage of the time that the signs of the slopes of the curves agree. This measure is sometimes called the slope congruence. Although the resolution is not good, the slope congruence method is able to distinguish the two sets of curves in figure 7.4. The left column produces a slope congruence of 90% while the right column produces a slope congruence of only 70%. These numbers indicate that there are 1 out of the 10 intervals in the left column and 3 out of 10 intervals in the right column in which the slopes of the curves are in the opposite direction. The Pearson product-moment correlation coefficient and the slope congruence methods produce measures of the overall temporal similarity between two curves. For this reason they are not the most sensitive measures if the temporal similarity is important during only a portion of the curves. In this case, one could employ discrete value techniques to measure temporal similarity. For instance, one could calculate the difference in the time to peak values. If the magnitude of the slope difference is important, the curves can be differentiated and then one curve can be subtracted from the other. Local maximum and minimum values would indicate when there is increased disagreement between the slopes of the curves. In this case it is possible to have a peak value even if both curves are in the same direction. One of the curves would be increasing (or decreasing) rapidly while the other is increasing (or decreasing) slowly.

Two curves can have a very high correlation value yet be separated by a large offset. As long as the curves are parallel, the correlation will be high. In order to assess differences in magnitude, one could subtract one of the curves from the other. This produces a difference curve in which peak differences could be identified or average differences calculated. If the curves cross during the period being analyzed, it is more appropriate to calculate the average absolute differences in the curves.

Example 7.3

Use the values for curve 1 and curve 2 shown in the table to calculate the average difference and the average absolute difference between the curves. Note the substantial difference between these two measures of magnitude differences.

Time	Curve 1	Curve 2
0.00	0.00	0.00
0.10	0.01	0.40
0.20	0.04	0.95
0.30	0.28	1.00
0.40	1.00	0.90
0.50	0.90	0.50
0.60	0.75	0.20
0.70	0.46	0.00
0.80	0.20	0.00
0.90	0.10	0.00
1.00	0.03	0.00

	Difference	Absolute difference
	0.00	0.00
	−0.39	0.79
	−0.91	1.86
	−0.72	1.72
	0.10	0.80
	0.40	0.10
	0.55	0.35
	0.46	0.46
	0.20	0.20
	0.10	0.10
	0.03	0.03
Average	−0.02	0.58

Correlograms

A plot of the cross-correlation values at each lag is called a correlogram. Such plots can be useful for extracting noisy signals, synchronizing signals, or finding hidden frequencies within a signal. Radar and sonar use correlograms to determine the distance of a remote object. A "chirp" signal is transmitted from a radar; it bounces off the object of interest and returns to its original position, where a receiver records the returning signal. We can calculate the distance after measuring the time between transmission and reception. The problem is that a significant amount of noise is introduced to the echoed signal and the original chirp may not be recognizable (figure 7.6). Cross-correlations between the transmitted and echoed signals will peak at the lag in which the signals line up. Figure 7.6 also shows the **correlogram** between a clean chirp signal (transmitted signal) and the same chirp signal with random noise introduced (echoed signal). The noise nearly hides the chirp, yet the correlogram shows a distinct peak cross-correlation at a zero lag. This indicates that the two signals are synchronized. If these signals were from a radar, the peak would occur at a particular lag that could be transformed into a transmission time through multiplying by the sampling period. The distance could be determined through multiplying half of the transmission time by the transmission velocity.

A similar process has been used to measure the conduction velocity in a muscle (Li and Sakamoto, 1997). The propagation wave is measured at more than one location on the muscle, and the delay is determined through cross-correlation of the signals. The peak cross-correlation value occurs at the lag where the signals have the greatest similarity. Conduction velocity can then be calculated from this lag time and from knowledge of the interelectrode distances.

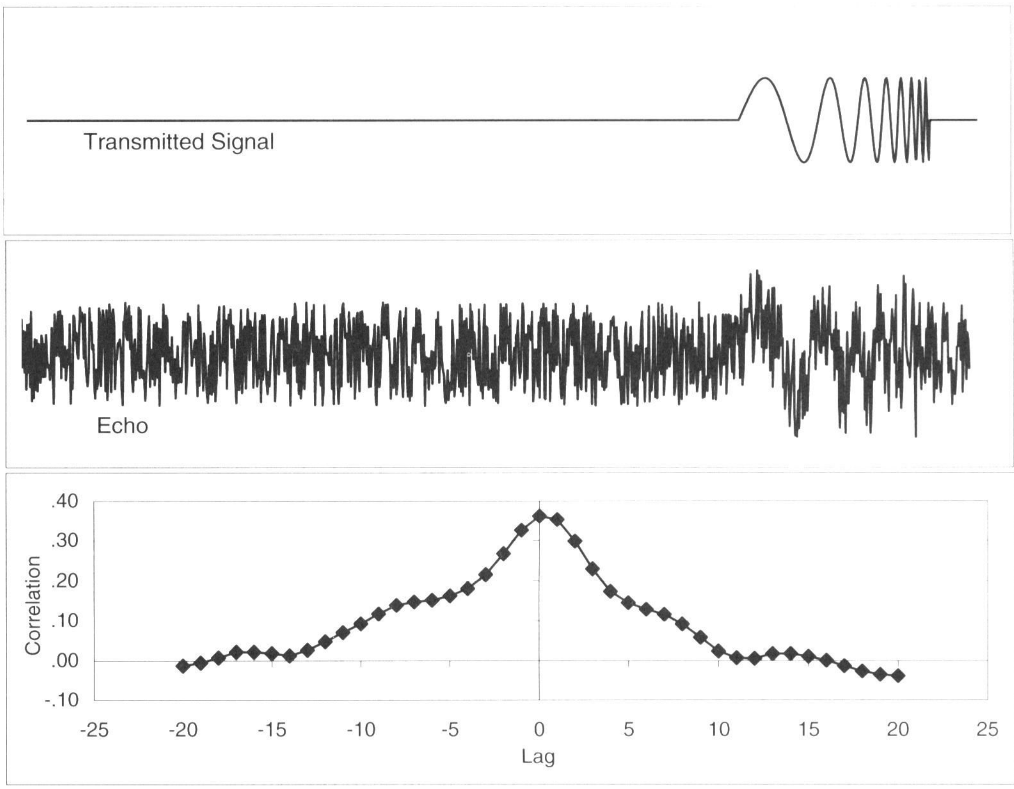


Figure 7.6 Correlogram of a chirp. The top curve is a "chirp" signal sent by a radar. The middle curve is the same chirp after it has been echoed off the object of interest and random noise has been introduced. The bottom curve is a correlogram showing the relationship between the signal and the echo at different lags.

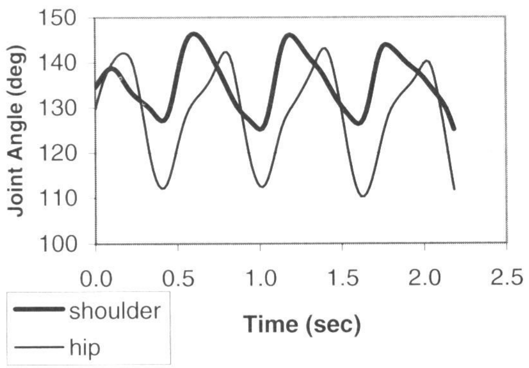


Figure 7.7 Shoulder-hip angle relationship. Hip and shoulder angles of a dog walking on a treadmill at 1.3 m/s. Approximately three cycles of data are shown.

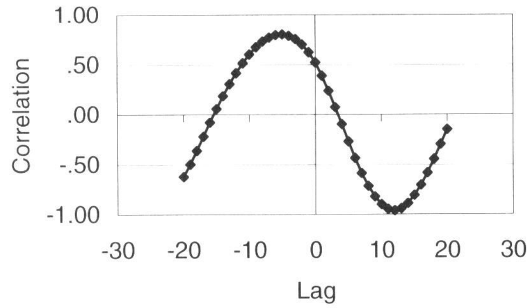


Figure 7.8 Shoulder-hip correlogram. A correlogram produced from the cross-correlation between shoulder and hip angles of a dog walking on a treadmill. Peak correlations occur when the shoulder and hip curves line up with each other. The lag at the peak value indicates the phase difference between the shoulder and the hip.

Cross-correlation can provide a cleaner picture of the timing between two signals than discrete point analysis can. Figure 7.7 shows the hip and shoulder angles of a dog walking at 1.3 m/s on a treadmill. These data were hand digitized at a frame rate of 60 Hz. It is difficult to assess the differences in timing between these two joints by evaluating discrete points. The hip seems to reach maximum extension (local maximums) later than the shoulder joint. On the other hand, a look at peak flexion (local minimums) suggests that the hip and shoulder are nearly synchronized. A correlogram evaluates entire curves rather than discrete points on the curve. Figure 7.8 shows the correlogram between the shoulder and hip joint angles. The peak cross-correlation occurs at a lag of –5. This indicates that there is a phase shift of –83 ms (–5/60 s) between the hip and the shoulder joint angles.

Example 7.4

lag	*r*
-6	0.139
-5	0.148
-4	0.152
-3	0.163
-2	0.181
-1	0.215
0	0.267
1	0.326
2	0.362
3	0.353
4	0.298
5	0.229
6	0.174
7	0.146
8	0.129

	Difference	Absolute difference
	0.00	0.00
	–0.39	0.79
	–0.91	1.86
	–0.72	1.72
	0.10	0.80
	0.40	0.10
	0.55	0.35
	0.46	0.46
	0.20	0.20
	0.10	0.10
	0.03	0.03
Sum	–0.18	6.41

The data in the table represent a correlogram between two electrodes attached to a nerve fiber. Calculate the nerve conduction velocity if the electrodes are spaced 120 mm apart and each lag represents 1 ms.

There is a lag of 2 between the signals measured at the two electrode sites. Since each lag is 1 ms, the total time between the signals is 2 ms. The propagated wave takes 2 ms to travel 120 mm and thus the velocity is 120 / 2 = 60 mm/ms or 60 m/s.

Autocorrelograms

A correlogram produced from the autocorrelation is called an autocorrelogram. Figure 7.9 shows correlograms for both the shoulder and the hip joints. These plots show peak correlations at lags of approximately 37 data points. This indicates that as the curve is slid past a copy of itself, the fluctuations in the copy will line up with the original after being shifted 37 data points. The autocorrelogram will continue to have peaks every 37 data points for as long as the lags are calculated. This indicates a cyclical period of 617 ms (37/60 s) or a walking frequency of 1.62 Hz. The same information is obtained using either the shoulder or the hip correlograms. Autocorrelograms can provide much information about a time series, and considerable experience is needed in their interpretation. Such items as nonstationarity, seasonal fluctuation, randomness, alternation, and short-term correlation can be determined from an autocorrelogram of a time series (Chatfield, 1984).

The autocorrelation function has been used to measure conduction velocity in muscles by means of a single channel of data (Spinelli, Felice, Mayosky, Politti, and Valentinuzzi, 2001) as opposed to the two channels required with use of the cross-correlation technique. The difference signal is obtained from two needle electrodes, and autocorrelation techniques are utilized to estimate the conduction velocity in periods as short as 0.3 s.

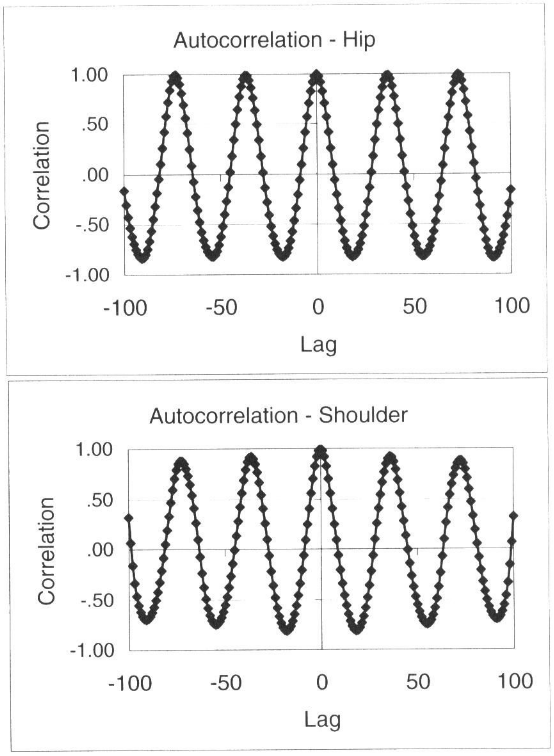


Figure 7.9 Shoulder and hip autocorrelations. Correlograms produced from the autocorrelation of the shoulder and hip angles of a dog walking on a treadmill. Peak correlations occur when a cycle lines up with a subsequent cycle. The inverse of the time between peak values indicates the cycle frequency.

Cross-Correlation As a Method for Estimating Spectral Content

Thus far in the chapter, the cross-correlation function has been used as a measure of temporal similarity, as an estimation of the time lag between signals, as a way to find periodicity within a signal, and as a way of discovering signals hidden within a signal (as in the radar example). This last application deserves further exploration. Previous examples correlated two signals that had both been collected from instruments. But it is not necessary for both signals to have been collected from instruments; one of the signals could be created from a function. For instance, the hip joint angle of a dog walking (figure 7.7) could be correlated with a sine wave (equation 7.7) of the same duration. In this case the cross-correlation can indicate how much of the sine wave of frequency f is contained in the signal. The signal could be out of phase with the sine wave, so a complete analysis would also include a cross-correlation between the signal and a cosine wave of frequency f.

$$\mathrm{sw}(t) = A\sin(2\pi ft) \tag{7.7}$$

where sw(t) is the sine wave, A is the amplitude, f is the frequency, and t is the time.

To illustrate this point, a signal h(t) was created using the following formula:

$$\mathrm{h}(t) = A_1\sin(2\pi f_1 t) + A_2\sin(2\pi f_2 t) + A_3\sin(2\pi f_3 t) \tag{7.8}$$

where $f_1 = 1, A_1 = 1; f_2 = 3, A_2 = 1 / 3 = 0.33$; and $f_3 = 5, A_3 = 1 / 5 = 0.20$.

These are the first three components of a square wave as calculated by adding sine waves of increasing frequency. The left column of figure 7.10 shows the time series graph of this formula. Cosine and sine waves of increasing frequency were created and graphed in the second column in figure 7.10.

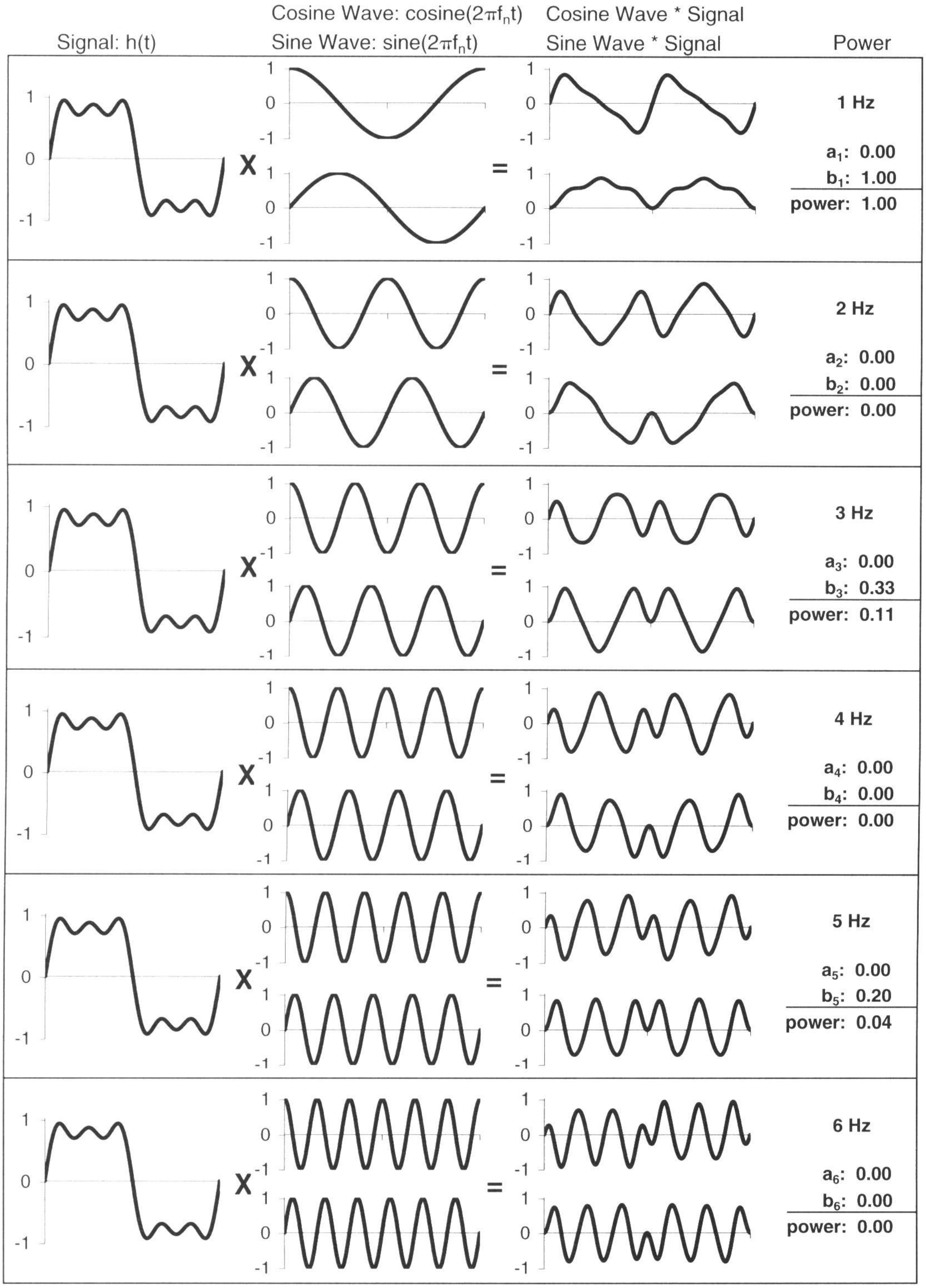


Figure 7.10 Calculation of Fourier coefficients using cross-correlation. The signal in the left column $h(t) = \sin(2\pi t) + 0.3\sin(2\pi 3t) + 0.2\sin(2\pi 5t)$ is cross-correlated with cosine and sine waves to obtain the Fourier coefficients (a_n and b_n, respectively). See text for details.

A modified version of equation 7.1 was used to calculate the cross-correlation between the signal and the cosine wave and the sine wave for each frequency (n). The cross-correlation was simply divided by the number of data points (N) and then multiplied by 2 (equation 7.9). This modification allows the cross-correlation value to correspond to the amplitude of the cosine or sine wave.

$$r_{xy} = \frac{2}{N}\sum_{i=0}^{N-1} x_i y_i \tag{7.9}$$

Since the signal was composed of 1-, 3-, and 5-Hz sine waves, all of the a_n coefficients (from the cosine wave) are zero, and the only b_n coefficients (from the sine wave) that are non-zero are at 1, 3, and 5 Hz. Note that the b_n coefficients are 1.00, 0.33, and 0.20 for the 1-, 3-, and 5-Hz sine waves, respectively, and that these coefficients correspond to the amplitudes of the sine waves that composed the original signal h(t). It is possible to decompose any signal into constituent sine and cosine waves of varying amplitude.

The a_n and b_n coefficients were calculated by cross-correlating the signal with sine and cosine waves of varying frequency (n) using equation 7.9. These coefficients are called **Fourier coefficients** after the French mathematician Jean Baptiste Joseph, Baron de Fourier (1768-1830). By squaring the a_n and b_n coefficients and then adding them together, one obtains the power for the frequency n. A plot of the power by frequency is generally known as a **power spectrum.** There are better ways to calculate the Fourier coefficients for a given signal, but this method has the advantage that it is intuitive. Chapter 9 describes power spectrums in more detail. The power spectrum of the signal represented by equation 7.8 is plotted in figure 7.11 for the frequencies from 1 to 6 Hz. The plot indicates the frequencies that are present in the signal. This method may be used for any time-varying signal, not just one composed of sine waves.

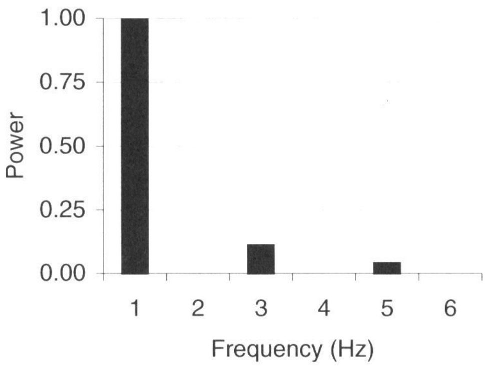


Figure 7.11 Power spectrum of a complex sine function. The power spectrum is of the signal h(t) = sin($2\pi t$) + 0.33sin($2\pi 3t$) + 0.2sin($2\pi 5t$). The powers were obtained by summing the square of the cosine and sine amplitudes of the Fourier coefficients.

Knowledge of the spectral content of a signal is important because different frequencies may have different biological significance. Shorten and Winslow (1992) used an efficient algorithm called a Fast Fourier Transform (FFT) to calculate the power spectrums from leg accelerations during running. The methods were used to determine which acceleration frequencies were present during running. The authors identified three frequency ranges (4-8 Hz, 12-20 Hz, and 60-90 Hz) in an accelerometer attached to the leg. The low frequencies were the result of muscular activity; the middle frequencies were the result of the foot striking the ground; and the high-frequency range resulted from resonance of the accelerometer attachment. The authors were most interested in the midfrequency range because of the potential for injury caused by the impact.

Other analysis and processing techniques such as data filtering can be built from knowledge of these frequency domain methods. For instance, data filtering can be accomplished through removal of particular frequencies from the data. Electrical equipment can contaminate signals by producing frequencies around 60 Hz. The signal can be transformed into the frequency domain; frequencies around 60 Hz could be eliminated or reduced; and then the signal could be transformed back into the time domain. Fourier filters eliminate the unwanted frequencies in the frequency domain; digital filters eliminate the frequencies in the time domain without transformations.

Matched Filters

Matched filters can be used whenever a known signal is present within a more complex signal or a corrupted signal. In an earlier example, the chirp from a transmitter was compared to the returning signal that was bounced off an object. The signal of interest, or template, is compared to the complex signal, and a high correlation indicates a "match." The template that is cross-correlated with the complex signal can be a sine wave or a cosine wave, as indicated in the section on spectral content, or it can be any other function. It can even be a portion of a curve. For instance, **motor unit action potential** (MUAP) **decomposition** can utilize cross-correlation techniques to identify the location of individual MUAPs. Low-force electromyographic (EMG) recordings are composed of several motor units, each firing at a relatively consistent

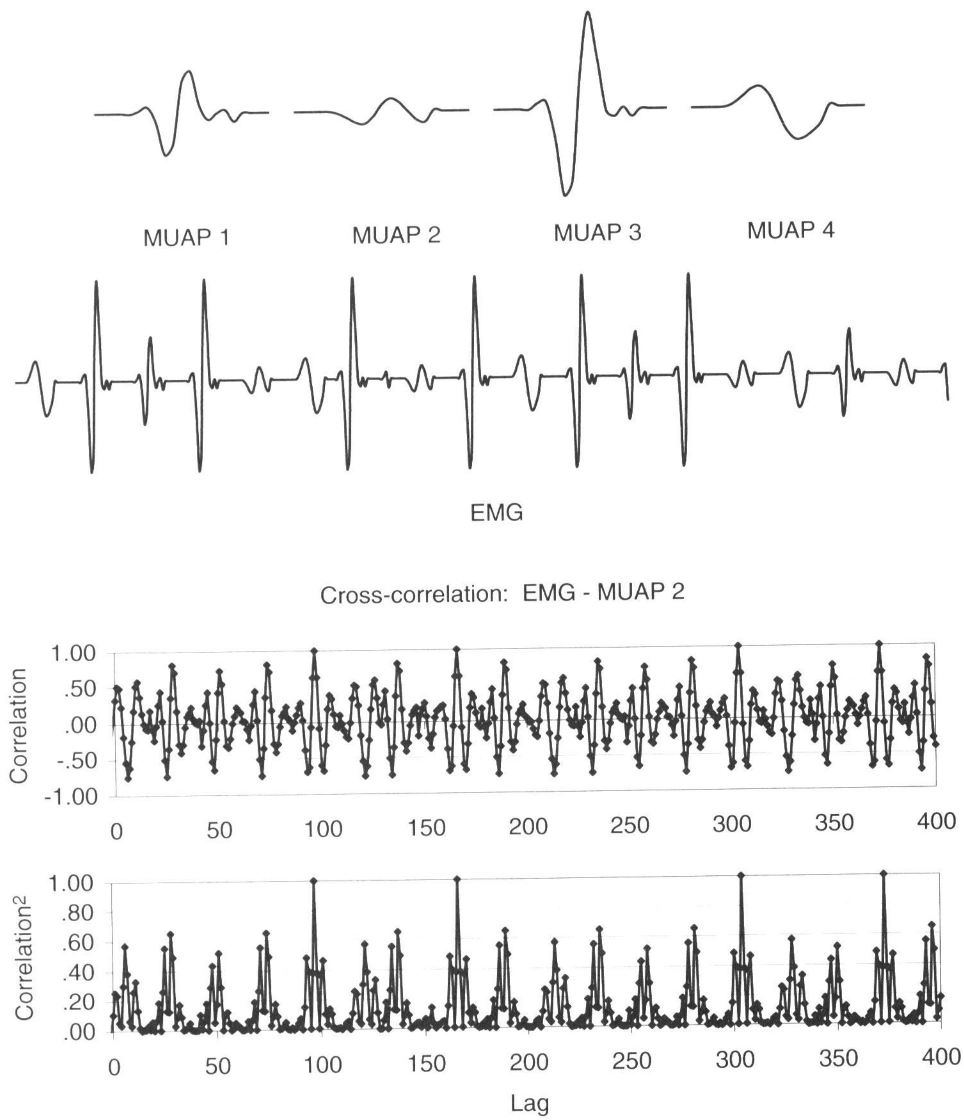


Figure 7.12 Identification of waveforms in a complex signal. Schematic of waveforms of four individual motor unit action potentials (MUAP) and an EMG signal created from multiple firings of the individual MUAP waveforms. Correlograms display MUAP 2 cross-correlated with the EMG signal. Firing of this motor unit occurs when the correlation approaches 1.0. The bottom correlogram shows that squaring the correlation coefficient can make identification of the firing easier.

rate. The EMG is digitized at high sampling frequency, and the first clear occurrence of each MUAP is manually extracted and used as a template. Several motor units are typically recorded at the same time, but each has its own unique waveform (see the four MUAP waveform templates at the top of figure 7.12). Thus, cross-correlating each waveform template with the EMG allows one to identify each firing of the motor unit within the EMG. It is then possible to reconstruct the MUAP trains by joining the multiple occurrences of each waveform together. This algorithm is referred to as a **matched filter** because it allows the template to be matched to subsequent occurrences of the same general shape (Karu, 1995).

Figure 7.12 shows a schematic waveform of four individual motor unit action potentials (MUAP). The EMG is composed of multiple firings of these four motor units. It is often useful to calculate statistics such as firing frequency and the variability of the firing frequency of each motor unit (DeLuca, 1983). In order to calculate these statistics, it is necessary to decompose the EMG into the individual MUAP firings. The correlograms in figure 7.12 show the second MUAP cross-correlated with the EMG in an attempt to identify when the second MUAP is firing. Correlations that approach 1.0 indicate a likely match between the template MUAP waveform and a firing of that motor unit in the EMG. Four such firings are indicated by the correlogram. The bottom correlogram shows that the correlations can be squared to give a clearer picture of when the motor unit is firing.

Summary

The analysis of time series curves consumes a relatively large portion of a research biomechanist's time. To be successful, the researcher must be familiar with both the data that have been collected and the possible analysis techniques. A time series data set can be correlated with another data set (cross-correlation), with itself (autocorrelation), with a function such as a sine wave, or with a template such as a known waveform (matched filter). In its simplest form the cross-correlation measures the similarity between two signals. It can also be used to determine differences in phase, extract hidden signals, and reveal frequency content. The purpose of this chapter was to acquaint the reader with these types of analysis techniques. Understanding the basis for cross-correlation can be useful in itself but can also lead to an intuitive feel for more advanced analysis techniques such as Fourier analysis, frequency filtering, and wavelet analysis.

Work Problems

1

Using the tables in example 7.1, shift curve 2 by 0.2 s and by 0.4 s and recalculate the correlations using equation 7.1.

2

Show that the average of three correlation coefficients (.70, .80, and .90) is not .80 when the Fisher Z-transformation is used to convert the coefficients to *z*-scores prior to taking the average. The average *z*-score can be transformed back into a correlation by using the following formula:

$$r = \frac{\exp(2z) - 1}{\exp(2z) + 1}$$

where r is the average correlation coefficient and z is the averaged z-score.

Suggested Readings and Other Resources

Books and Articles

Karu, Z.Z. (1995). *Signals and systems made ridiculously simple.* Cambridge, MA: ZiZi Press.

Oppenheim, A.V., and Schafer, R.W. (1989). *Discrete-time signal processing.* Englewood Cliffs, NJ: Prentice Hall.

Proakis, J.G., and Manolakis, D.G. (1988). *Introduction to digital signal processing.* New York: Macmillan.

Transnational College of LEX. (1995). *Who is Fourier? A mathematical adventure.* Translated by Alan Gleason. Belmont, MA: Language Research Foundation.

Web Sites

www.falstad.com/fourier/

www.stats.gla.ac.uk/steps/glossary/time_series.html

www.statsoftinc.com/textbook/sttimser.html

References

Chatfield, C. (1984). *The analysis of time series.* New York: Chapman & Hall.

DeLuca, C.J. (1983). Myoelectrical manifestations of localized muscular fatigue in humans. *Critical Reviews in Biomedical Engineering,* 11(4):251-279.

Derrick, T.R., Bates, B.T., and Dufek, J.S. (1994). Comparative evaluation of time-series data sets using the Pearson product-moment correlation coefficient. *Medicine and Science in Sports and Exercise,* 26(7): 919-928.

Derrick, T.R., Caldwell, G.E., and Hamill, J. (2000). Modeling the stiffness characteristics of the human body while running with various stride lengths. *Journal of Applied Biomechanics,* 16(1):36-51.

Hamill, J., Caldwell, G.E., and Derrick, T.R. (1997). A method for reconstructing digital signals using Shannon's sampling theorem. *Journal of Applied Biomechanics,* 13:226-238.

Karu, Z.Z. (1995). *Signals and systems made ridiculously simple.* Cambridge, MA: ZiZi Press.

Li, L., and Caldwell, G.E. (1999). Coefficient of cross correlation and the time domain correspondence. *Journal of Electromyography and Kinesiology,* 9:385-389.

Li, W., and Sakamoto, K. (1997). Distribution of muscle fiber conduction velocity of M. biceps brachii during voluntary isometric contraction with use of surface array electrodes. *Applied Human Science: Journal of Physiological Anthropology,* 15(1):41-53.

Otnes, R.K., and Enochson, L. (1978). *Applied time series analysis.* New York: Wiley.

Shorten, M.R., and Winslow, D.S. (1992). Spectral analysis of impact shock during running. *Journal of Sport Biomechanics,* 8:288-304.

Spinelli, E., Felice, C.J., Mayosky, M., Politti, J.C., and Valentinuzzi, M.E. (2001). Propagation velocity measurement: Autocorrelation technique applied to the electromyogram. *Medical and Biological Engineering and Computing,* 39(5):590-593.

Stergiou, N., Bates, B.T., and James, S.L. (1999). Asynchrony between subtalar and knee joint function during running. *Medicine and Science in Sports and Exercise,* 31(11):1645-1655.